



Formation Flying Near Sun-Earth L2 Point Analysis and Design



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Abstract

A few space agencies are planning missions to the vicinity of the Sun – Earth L2 libration point, some involving a system of spacecraft forming a distributed telescope, configured on a virtual surface about the hub and along the focal length. In the next 10 years, a complete understanding of the dynamics of such objects is necessary for upcoming missions such as MAXIM, Stellar Imager (NASA) or DARWIN (ESA).

Upon examining the dynamics of Earth-Sun system, this study focuses on the Circular Restricted Three Body (CRTB) problem. Based on previous studies of the relative motion of Earth Orbiting bodies, a Lissajous orbit is investigated. Different initial conditions are analyzed to find the most stable relative motion. The solution of two symmetric, quasi-circular orbits is presented.

Analysis Method

With Circular Earth Orbit Relative Motion

Hill's Equations of the Relative motion



Solutions = $f(t, dx(0), dy(0), dz(0), dVx(0), dVy(0), dVz(0))^1$



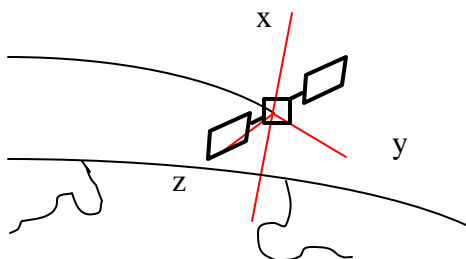
Study of the effects of the variation
of each initial condition on the relative motion



find the stable motions

Results: Initial conditions are chosen such that the difference in Orbital period is zero. This represents a relationship between $dx(0)$ and $dVy(0)$

Initial conditions $dy(0), dVx(0), dz(0), dVz(0)$ result in periodic motion. The analysis shows that the z motion is uncoupled in Hill's Equations. The resulting relative motion between the two spacecraft for $dVx(0)$ is an ellipse (difference in eccentricity) with these restrictive assumptions.



Stellar Imager

The Stellar Imager (SI) is a mission to understand the various effects of magnetic fields of stars, the dynamos that generate them, and the internal structure and dynamics of the stars in which they exist. The ultimate goal is to achieve the best-possible forecasting of solar activity on time scales ranging up to decades, and an understanding of the impact of stellar magnetic activity on astrobiology and life in the Universe

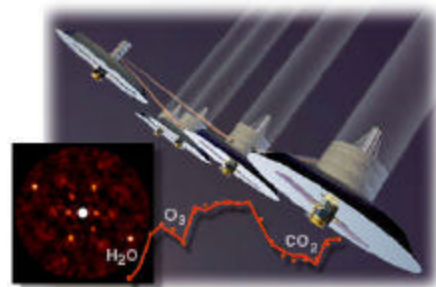
MAXIM

The MAXIM concept utilizes X-ray interferometry to achieve micro-arcsecond angular resolution. A NASA mission built around this concept could achieve resolution as fine as 300 nano-arcseconds. This resolution is 3,000 times finer than VLBI and 300,000 times finer than HST

DARWIN

Darwin will use a flotilla of six space telescopes, each of which will be at least 1.5 meters in diameter. They will work together to scan the nearby Universe, looking for signs of life on Earth-like planets. This is a daunting challenge and will require a number of technological innovations before the mission launches in the middle of the next decade, observing in the infrared because life on Earth leaves its mark at these wavelengths.

TPF
Project



The Circular Restricted Three-Body Problem

Two massive particles move in circle around their center of mass and attract (but are not attracted by a third particle of infinitesimal mass. The problem is to determinate its possible movements.

Equations of motion :

$$\begin{aligned} \ddot{x} - 2\dot{y} &= \frac{\partial U}{\partial x} & r_1 &= \sqrt{(x-\mu)^2 + y^2 + z^2} \\ \ddot{y} + 2\dot{x} &= \frac{\partial U}{\partial y} & r_2 &= \sqrt{(x-1+\mu)^2 + y^2 + z^2} \\ \ddot{z} &= \frac{\partial U}{\partial z} & \mu &= \frac{M_2}{M_1 + M_2} \end{aligned}$$

With $U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$
(Jacobi)

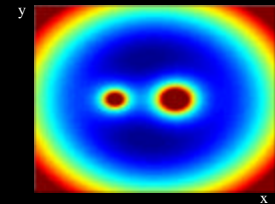
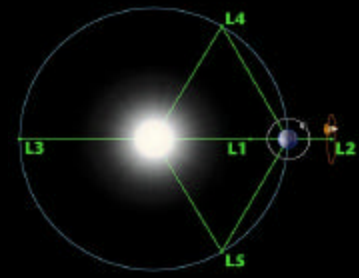
Time is in units of $1/n$, and x, y, z, r_1, r_2 are in units of Sun-Earth distance

n = rotation speed of the earth

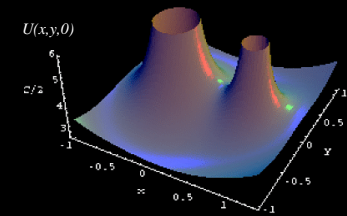
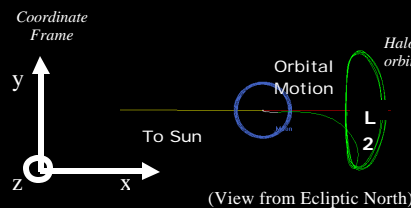
5 points are found in the xy plane where the partial derivatives of the Jacobi vanish. We also demonstrate that at these five points, no resultant force acts on the particle. They are equilibrium points of the system known as Lagrange points or libration points.

L1, L2, L3 are unstable and L4, L5 are stable.

In the set of all solutions of the problem, there are periodic orbits around every libration point. The most interesting ones are Lissajous and Halo orbits (elliptical shape like orbits)



Jacobi curves in the xy plane



Relative Motion in Lissajous Orbits

Linear solution of the CRTB Problem around L2 :

$$x(t) = x(0) \cos \omega_{xy} t + (1/k) y(0) \sin \omega_{xy} t$$

$$y(t) = y(0) \cos \omega_{xy} t + k x(0) \sin \omega_{xy} t$$

$$z(t) = z(0) \cos \omega_z t + [Vz(0)/\omega_z] \sin \omega_z t$$

Relative motion :

$$x_t(t) - x_h(t) = dx(0) \cos \omega_{xy} t + (1/k) dy(0) \sin \omega_{xy} t$$

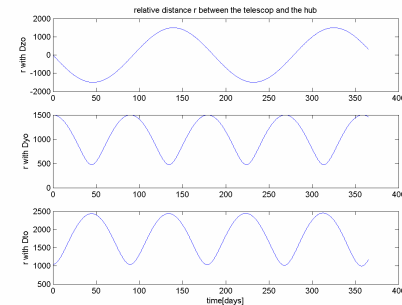
$$y_t(t) - y_h(t) = dy(0) \cos \omega_{xy} t + k dx(0) \sin \omega_{xy} t$$

$$z_t(t) - z_h(t) = dz(0) \cos \omega_z t + [dVz(0)/\omega_z] \sin \omega_z t$$

$$\begin{aligned} dx(0), dy(0), \\ dz(0), dVz(0) \end{aligned}$$



Periodic Motions



Minimization of the variation of the relative distance for :

$$dx(0)$$

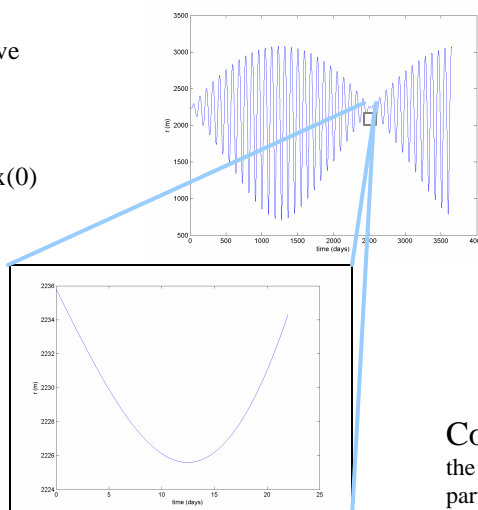
$$dy(0) = \pm k \cdot dx(0)$$

$$dz(0) = \pm \sqrt{k^2 - 1} \cdot dx(0)$$

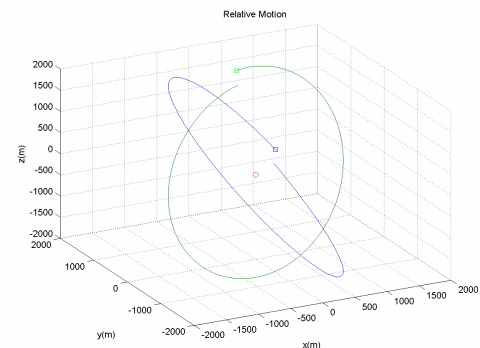
$$dVz(0) = \pm \omega_z \cdot dz(0)$$

particular solution

Small variation of the distance between the two spacecraft for 22 days



⇒ Two circular orbits



Conclusion : Relative distance between the telescope and the hub is stable for approximately 20 days. With control, these particular orbits are very low-cost to maintain.